# The Poisson Distribution

## 1.1 Recall: The binomial distribution

**DEFINITION Binomial Distribution**

An r.v. has the **binomial distribution** with parameters and has a p.f. as follows:

## 1.2 Relation between BD and Poisson Distribution

泊松分布是二项分布在n较大、p较小时的极限

EXAMPLE Customer Arrivals

A store owner observes that customers arrive at his store at a rate of 4.5 customers per hour on average. He wants to find the distribution of the actual number of customers who will arrive during a particular one-hour period later in the day.

As an approximation, he believes that the arrival rate per second is . He also assumes that during each second either 0 or 1 customer will arrive, thus the probability of an arrival during any second is .

First, he feeds the parameters and into the p.f. of BD. Obviously, the calculation is too cumbersome. However, he realizes that the successive values of are closely to each other because changes in a systematic way as increases. So he computes

Let , then we get the following pattern

Continuing the pattern yields

To obtain a p.f., let . Thus

From the well-known calculus result,

we get

## 1.3 Poisson distribution

**DEFINITION Poisson distribution**

A random variable has the **Poisson distribution with mean**  if the p.f. of is as follows:

**THEOREM Mean of PD**

The mean of PD is .

PROOF is given by the following infinite series

Since the term corresponding to in this series is 0, we can omit this term and can begin the summation with the term for . Therefore,

Let , we yield

**THEOREM Variance of PD**

The variance of the Poisson distribution with mean is also .

PROOF

Since , it follows that . Therefore,

## 1.4 Poisson process

**DEFINITION Poisson Process**

A Poisson process with rate per unit time is a process that satisfies the following two properties:

1. The number of arrivals in every fixed interval of time of length has the Poisson distribution with mean .
2. The numbers of arrivals in every collection of disjoint time intervals are independent.